Comparison of main magnetic force computation methods for noise and vibration assessment in electrical machines

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In the vibro-acoustic analysis of electrical machines, the Maxwell Tensor in the air-gap is widely used to compute the magnetic forces applying on the stator. In this paper, the Maxwell magnetic forces experienced by each tooth are compared with different calculation methods such as the Virtual Work Principle based nodal forces (VWP) or the Maxwell Tensor magnetic pressure (MT) following the stator surface. Moreover, the paper focuses on a Surface Permanent Magnet Synchronous Machine (SPMSM). Firstly, the magnetic saturation in iron cores is neglected (linear B-H curve). The saturation effect will be considered in a second part. Homogeneous media are considered and all simulations are performed in 2D. The technique of equivalent force per tooth is justified by finding similar resultant force harmonics between VWP and MT in the linear case for the particular topology of this paper. The link between slot’s magnetic flux and tangential force harmonics is also highlighted. The results of the saturated case are provided at the end of the paper.

Index Terms—Electromagnetic forces, Maxwell Tensor, Virtual Work Principle, Electrical Machines.

I. INTRODUCTION

In electrical machines, the study of noise and vibrations due to magnetic forces first requires the accurate calculation of Maxwell stress distribution which depends on the time and space distribution of the magnetic flux density. Indeed a very small magnetic force harmonic can induce large acoustic noise and vibrations due to a resonance with a structural mode of the stator. The magnetic flux can be determined everywhere in the machine with a Finite Element Analysis (FEA), only in the air-gap and windings with semi-analytical methods such as Sub-Domain Model (SDM) [1], or only in the middle of the air-gap using the permeance magneto-motive force (PMMF).

In order to compute magnetic forces, a various range of methods can be found in the literature including: fictive magnetic currents and magnetic masses methods [2], energy methods [3], Maxwell Tensor (MT) methods [4], and Virtual Work Principle adapted to Finite Element (VWP) [5]. Then a compatible force computation method for the vibro-acoustic objectives must be chosen. As shown in the Table I the two flagship methods are the VWP and the MT but the preference for one method or another is not clearly justified in the documentation and articles linked to these software (see column “Ref.” of Table I).

The discussion of local magnetic pressure is still on going [24] [25] [26] and should be the aim of further work. The VWP is built to account for local magnetic behavior, while the Maxwell tensor gives the momentum flux across any surface, closed or not [24]. Historically the Maxwell Tensor has been used by electrical machine designers to accurately compute global quantities such as electromagnetic torque, which is the moment of the global magnetic force applied on a cylinder surrounding the rotor on its axis of rotation. Under the common form the MT cannot be rigorously related to a local magnetic pressure but it is often used in vibro-acoustic studies to obtain the local force distribution in the air gap which applies on the stator structure [8] [11] [15] [20]. However such a method can have strong limitations depending on the geometry as shown by [17].

Consequently, a common method to compute the magneto-mechanical excitations is to apply one integrated force per stator’s tooth. It corresponds to the “Lumped Force Mapping” method proposed in [15]. A comparable integrated force per
The nodal forces with Virtual Work Principle

A. Formulas & implementation

Let \( \mathbf{H} \) be the magnetic field, \( \mathbf{B} \) the magnetic flux density, \( H_i \) and \( B_i \) their respective components in cartesian frame. The nodal force expression proposed in [5] is based on an equivalence between the magnetic co-energy variation and the force applied on a solid determined by the domain \( \Omega \), such that the force amplitude in the direction \( s \in \{x, y, z\} \) is:

\[
F_s = \frac{\partial}{\partial s} \int_{\Omega} \mathbf{B} \cdot d\mathbf{H} \, d\Omega
\]  

(1)

Applying this equation to a mesh element \( e \):

\[
F_s = \int_{\Omega} \left( -\mathbf{B}^T \cdot J^{-1} \cdot \frac{\partial J}{\partial s} \cdot \mathbf{H} + \int_{0}^{H} \mathbf{B} \cdot d\mathbf{H} \, |J^{-1}| \frac{\partial |J|}{\partial s} \right)
\]  

(2)

with \( J \) the Jacobian matrix of the element \( e \). The derivatives of \( J \) can be determined knowing the type of element (triangular, rectangular, quadrilateral, tetrahedron ...). With a linear case, the integrand of \( \mathbf{B} \) can be simplified as follow:

\[
\int_{0}^{H} \mathbf{B} \cdot d\mathbf{H} = \int_{0}^{H} \mu \mathbf{H} \cdot d\mathbf{H} = \frac{\mu}{2} |\mathbf{H}|^2
\]  

(3)

with \( \mathbf{H} \) constant in each element. Thus the natural way to implement the VWP algorithm is to loop on each element, compute the previous formulas and add the element contribution to its nodes. For a given node \( i \):

\[
F_s^i = \sum_{\forall e \in \mathcal{E}} \int_{e} \left( -\mathbf{B}^T \cdot J^{-1} \cdot \frac{\partial J}{\partial s} \cdot \mathbf{H} + \frac{\mu}{2} |\mathbf{H}|^2 \, |J^{-1}| \frac{\partial |J|}{\partial s} \right)
\]  

(4)

B. Discussion

An overview of the magnetic force distribution according to Eq. (4) is presented on Fig. 2. A first observation is the concentration of nodal forces at the iron-air interface. This result is expected because of the linear homogeneous hypothesis media for the stator. In a linear electromagnetic media, the electromagnetic co-energy density is:

\[
\psi(x, B(x)) = \frac{1}{\mu} \frac{|\mathbf{B}|^2}{2}
\]  

(5)
Then the variation of energy density due to a virtual (or real) displacement \( p \) : \( x \in \mathbb{R}^2 \to p(x) \in \mathbb{R}^2 \) in case of weak-coupling (\( B(p(x)) \approx B(x) \)) is:

\[
\psi(p(x), B(x)) - \psi(x, B(x)) \approx (1/\mu(p(x)) - 1/\mu(x)) |B(x)|^2/2
\]

In the case of linear elasticity (small displacement) \( \exists h \in \mathbb{R}^2 \), \( p(x) = x + h \) with \( h \to 0 \) such that:

\[
\partial_n \psi(B) = \lim_{h \to 0} \frac{\psi(x + h, B) - \psi(x, B)}{h} = \lim_{h \to 0} \frac{1/\mu(x + h) - 1/\mu(x)}{h} |B(x)|^2/2 = \nabla(1/\mu) |B(x)|^2/2
\]

The virtual work principle theorem (not the VWP method applied to FEA, see \( [30] \) for explanations) allows to identify the magnetic bulk force:

\[
f_{\text{mag}} = \partial_n \psi(B) = \nabla(1/\mu) |B(x)|^2/2
\]  

(6)

Therefore, if the B-H curve is linear and \( \mu \) constant in the stator, then we have \( f_{\text{mag}} = 0 \). This demonstration is inspired from the work of \([31]\) who proposes to treat the interface problem with derivation in the distribution sense.

Another observation on the Fig. 2 is the concentration of the forces on the tip of each tooth. In order to investigate the accuracy of lumped force applied at the tooth tip, the Fig. 3 presents the displacement range of the barycenter of force \((x_G, y_G)\) for one tooth over one revolution. For each tooth, the barycenter is computed by summing the contribution of all nodes \( i \) of the tooth:

\[
(x_G, y_G) = \frac{\sum_i (F_i x_i / F_i)}{\sum_i F_i}
\]

(7)

with \( F_i = \sqrt{(F_{ix})^2 + (F_{iy})^2} \) the \( L^2 \) norm of the force applied on the \( i^{th} \) node and \((x_i, y_i)\) its Cartesian coordinates. On average, the center of force is close to the center of the tooth tip and under 2% of the total tooth height. The asymmetrical path and the difference between the average center and the tooth’s tip center are explained by the asymmetrical average tangential forces between both edges, which produce the electromagnetic torque.

The linear hypothesis also means that both magnetic co-energy and magnetic energy are equivalent. However, this energetic symmetry is lost with non-linear media (by example for permanent magnets) resulting in different force distribution between co-energy and energy based formulations. This problem has been highlighted in \([32]\), who proposed a solution based on a “reference” energy. A solution is also proposed in \([33]\), by demonstrating generalized MT formulas and applying it on a FEA mesh with a virtual work principle formulation.

III. MAXWELL TENSOR MAGNETIC PRESSURE

A. Formulas & implementation

The Maxwell stress tensor \( T \) is commonly defined with the following component:

\[
T_{i,j} = B_i H_j - \frac{\mu}{2} \delta_{ij} \sum_{k=1}^n |H_k|^2
\]

(8)

Defining a surface \( S \) around a volume \( V \), such that the divergence theorem can be applied, with \( n \) the outer normal to \( S \). Then the global force \( F \) applying on the volume \( V \) is:

\[
F = \int_V \nabla T \, dV = \int_S \left( (B, n) \mathbf{H} - \mu/2 |\mathbf{H}|^2 n \right) dS
\]

(9)

Applied to the ferromagnetic tooth surrounded by a dotted circular path on the Fig. 4 the force can be written under the form:

\[
\begin{align*}
F_n &\approx \frac{1}{2} \int_S |B_n|^2 - \mu_0 |H_t|^2 dS + \Gamma_n^{S \cap S'} \\
&\quad - \frac{1}{2} \int_{S'} |B_n|^2 - \mu |H_t|^2 dS' \\
F_t &\approx \int_S B_n H_t dS - \int_{S'} B_n H_t dS' + \Gamma_t^{S \cap S'}
\end{align*}
\]

(10)

with \( \mu \) the magnetic permeability, \( \mu_0 \) the void permeability, \( X_n \) and \( X_t \) the components of a vector field respectively.
projected on \( n \) and \( t \) (local vectors attached to a surface) and \( \Gamma^{S \cap S'} \) a gap term corresponding to the integration over the interface between the stator and the air. This form assumes the previous application of the divergence theorem is still valid when crossing a discontinuity of permeability which is the interface stator-air. It can be achieved using the mathematical distribution theory and an integration in the distribution sense.

However the common form used to compute forces applying on one tooth is:

\[
\begin{align*}
F_n & \approx \frac{1}{2} \int_S \left( \frac{|B_n|^2}{\mu_0} - \mu_0 |H_t|^2 \right) dS - \frac{1}{2} \int_{S'} \left( \frac{|B_n|^2}{\mu} - \mu |H_t|^2 \right) dS' \\
F_t & \approx \int_S B_n H_t dS - \int_{S'} B_n H_t dS'
\end{align*}
\]

assuming \( \Gamma^{S \cap S'} \) is null. Indeed, the numerical effect is negligible \([34]\) when the integration surface cross the interface at points such as \( E \) and \( F \) on Fig. 4 where the magnetic flux in the air is low. Adding the hypothesis \( \mu \gg \mu_0 \), the force expression is commonly reduced to:

\[
\begin{align*}
F_n & \approx \frac{1}{2} \int_S \frac{1}{\mu_0} |B_n|^2 - \mu_0 |H_t|^2 \ dS \\
F_t & \approx \int_S B_n H_t \ dS
\end{align*}
\]

In the vibro-acoustic context, simplifying assumptions are often added by neglecting the \( H_t \) terms which leads to:

\[
\begin{align*}
F_n & \approx \frac{1}{2} \int_S \frac{1}{\mu_0} |B_n|^2 \ dS \\
F_t & \approx 0
\end{align*}
\]

These simplifications are coherent with analytical methods like PMMF which only compute radial magnetic field. It may be justified by the small effect of tangential lumped forces on the structural modes excitation \([35]\). To the authors knowledge, these assumptions have not been rigorously proved when considering local magnetic pressure.

**B. Discussion**

At this point, the integrand in Equation (12) could be identified as a "local magnetic pressure", as represented on the Fig. 5. Moreover, the previous VWP results, which show that the forces are indeed concentrated at the iron-air interface, corroborate the existence of such "local magnetic pressure". But as mentioned previously, it implies to arbitrary neglect some parts of the integration surface. A rigorous application of the MT is discussed in \([26]\). Even under the integrated form, there are many discussions about the choice of the integration surface, and its high influence on the vibro-acoustic results is a well-known problem \([36]\) \([37]\). According to \([38]\), the best choice with triangular finite elements is to cross the middle of the closest edges around the moving part, as illustrated with a closed path around a rectangular targeted body on Fig. 6. It agrees with \([39]\) saying the integration surface should stay close to the targeted body. However, the construction of such a contour requires to find the correct edges and their neighborhoods. This operation increases the computation/implementation complexity especially with sharp or non-standard geometry and the precision gain might not be huge. It could explain why so many FEA software rely on the extrapolation of the middle air gap force to the lumped tooth model.

Regarding the semi-analytical methods, the magnetic field is not discretized such that the MT can be easily computed along any path. However, close to geometrical singularities (such as teeths corner) some unphysical Gibbs phenomenon can appear. To avoid this particular issue the MT is commonly computed at a certain distance of any interface (for example in the middle of the air-gap).

Concerning analytical methods (PMMF), only the normal magnetic field component in the air-gap is available, such that the MT computed in this context is equivalent to the Equation (13).
Fig. 7. Computing magnetic pressure in function of the angular position according to the VWP, the MT along the interface and the MT in the air-gap.

Although the local force density may be wrongly estimated, the MT ensures that integrated forces and torque are correctly conserved. It is coherent with the historical use of Maxwell Tensor by electrical machine designers to accurately compute torque. Moreover, [40] shows that approximation can be good enough to successfully predict the structural displacement using a MT local magnetic pressure with semi-analytical methods.

IV. METHODS COMPARISON: LINEAR CASE

A. Local magnetic pressure

Fig. 7 compares local magnetic pressure computation according to different methods on the electrical machine of Fig. 1. At first only the radial component is studied since it is the main contributor to magnetic force. Three different methods are compared: the VWP computation based on FEA with FEMM [29], a “MT Interface” applied along the interface stator-air based on both SDM and FEA simulations, and a “MT Air gap” applied at constant radius in the middle of the air-gap based on FEA. Details about the application of these methods are provided in the Table III. The three different MT methods rely on the Equation (12) without the integrand. As observed on both Fig. 7 and 8 methods do not match each other on teeth tip, especially close to the tooth corners (i.e. the points C and D). The magnetic pressure distribution highly depends on the choice of the distance between the integration path and the interface, and the choice of this distance is discussed afterwards.

Since the objective is the vibro-acoustic analysis, a logarithmic scale is more adapted because the acoustic amplitude A related to the magnetic force amplitude F is:

\[ A \propto 20 \log\left(\frac{F}{F_{\text{ref}}}\right) \]

with \( F_{\text{ref}} \) a reference force. In this paper case \( F_{\text{ref}} = 1 \) [N] is used. Thus the magnetic forces are represented in logarithmic scale on Fig. 8 for radial component and on the Fig. 9 for the tangential component.

The dependence of the MT with the distance to the interface is even more clear in front of the slots. Initially, the MT is based on a volume, which is reduced to surface through a numerical approximation. This approximation is fulfilled in a linear case with a very high permeability for the stator. Then the tangential magnetic component \( H_t = 0 \) and the normal flux \( B_n \) is nearly constant close to the interface, such that the MT tends to the following magnetic pressure \( P_{\text{mag}} \) approximation:

\[ P_{\text{mag}} \approx \frac{1}{2\mu_0} |B_n|^2 \]

which is accurate for linear, planar surface with \( \mu_{\text{stator}} \gg 1 \). However, the geometric singularities (tooth’s corners) does not satisfy these assumptions.

Since semi-analytical methods are used, the discretization effect is not studied here. An analysis of the mesh effect on the MT is proposed in [41] and further investigations about the accuracy of the MT depending on the integral surface are
available in [42]. Although the local magnetic pressure may be wrongly estimated, this method ensures that integrated forces and torque are correctly conserved as shown in the following sections.

B. Lumped force mapping

In this section, the previous magnetic pressures are integrated and compared for a vibro-acoustic purpose. First an example of the results with these "lumped forces" is presented on Fig. 10.

To better understanding the notion of "magnetic momentum" represented by the MT, additional versions of the MT lumped forces are computed in the following. The set of methods is summarized in the Table III.

### Table III

<table>
<thead>
<tr>
<th>Name</th>
<th>Force method</th>
<th>Magnetic field method</th>
</tr>
</thead>
<tbody>
<tr>
<td>MT Air-gap</td>
<td>Maxwell Tensor computed according to Equation (12) along the green curve</td>
<td>SDM with MANATEE or FEA with FEMM</td>
</tr>
<tr>
<td>MT Interface</td>
<td>Maxwell Tensor computed according to Equation (12) along the beige curve</td>
<td>SDM with MANATEE or FEA with FEMM</td>
</tr>
<tr>
<td>VWP</td>
<td>Virtual work principle applied to Finite Element according to Equation (4)</td>
<td>FEA with FEMM</td>
</tr>
<tr>
<td>MT-radial</td>
<td>Maxwell Tensor computed according to Equation (13) along the green curve</td>
<td>SDM with MANATEE</td>
</tr>
<tr>
<td>MT Tooth Tip</td>
<td>Maxwell Tensor computed according to Equation (13) along the green curve</td>
<td>SDM with MANATEE</td>
</tr>
</tbody>
</table>

The first global value which can be numerically compared is the magnetic torque applying on the stator in Table IV.

### Table IV

<table>
<thead>
<tr>
<th>Average torque</th>
<th>Value [N.m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MT Air-gap (FEA)</td>
<td>-8.3</td>
</tr>
<tr>
<td>MT Interface (FEA)</td>
<td>-9.2</td>
</tr>
<tr>
<td>VWP (FEA)</td>
<td>-9.2</td>
</tr>
<tr>
<td>MT Radial (SDM)</td>
<td>0</td>
</tr>
<tr>
<td>MT Tooth Tip (SDM)</td>
<td>-5.0</td>
</tr>
<tr>
<td>MT Air-gap (SDM)</td>
<td>-8.1</td>
</tr>
<tr>
<td>MT Interface (SDM)</td>
<td>-9.9</td>
</tr>
</tbody>
</table>

For each tooth, the integrated forces computed according to the different methods are represented on Fig. 11 for the radial component, on Fig. 12 for the tangential component and on Fig. 14 for the electromagnetic torque. The resulting frequencies $f_n$ of the force harmonics are even multiple of the fundamental frequency $f_s$, such as :

$$f_n = 2nf_s = (2n) \frac{N}{60} \approx (53.33)n \ [Hz], \ \forall n \in \mathbb{N}$$

This result is observed for all three methods on both figures after performing a Fast Fourier Transform (FFT) on the discrete temporal signals. The three methods give the same frequency content, only the magnitude of the harmonics differ. The value of $A$ according to (14) differs of less than 3% between the different methods for the radial component of Fig. 11.

Moreover, effect of numerical noise can be observed on the Fig. 13 for VWP (and to a lesser extent for the MT FEMM) through small non-physical harmonics. This noise is due to mesh discretization. Besides only three out of five methods really account for the tangential direction contribution: VWP, "MT Air-gap" and "MT Interface". Indeed only the MT methods which take into account the slot magnetic flux can capture correctly the tangential lumped force. Thus "MT-radial" and "MT Tooth Tip" are not appropriate to study the tangential excitations. Moreover, neglecting the tangential magnetic flux (with the "MT-radial") leads to overestimate the radial lumped force.
Since the components harmonics and amplitudes are very close, the following methods are nearly equivalent from a vibro-acoustic point of view: VWP, MT Interface, MT Air-gap and MT FEMM. Then the lumped force mapping can be used for vibro-acoustic prediction with similar tooth geometry. For example, [28] presented a successful prediction of the noise with lumped force mapping using the "MT Air-gap". The case of Table [II] has a good agreement between linear and non-linear simulations. In order to start a study about the saturation effect, the following section proposes to keep the same geometry with higher injection currents and higher remanent magnetization.

C. Saturation effect

In this section, the simulation is now performed with the same geometry but the injected current and the remanent magnetization of the rotor’s magnet are both increased. The modified magnetic and windings parameters can be found in the Table [V]. The FEMM simulation is performed with a non-linear B-H curve presented on the Fig. [15] such that the saturation can be computed. The most visible effect of the saturation is the recoil of forces inside the stator volume as shown on the Fig. [16]. Indeed, the saturation results in a drop of the local magnetic permeability inside the material. It creates a gradient of permeability across the media such that a magnetic bulk force (orthogonal to iso-permeability lines) is obtained according to the Eq. (6). Thus magnetic forces are not completely concentrated at the interface unlike the linear case. Though the barycenter of forces on Fig. [17] is pushed back into the stator compared to the Fig. [3] the highest point is approximately at 22% of the total tooth height, such that the application of the force at the tooth starts to be questionable. Moreover the "MT Interface" approximation of the barycenter position is distorted by the presence of bulk forces.

<table>
<thead>
<tr>
<th>New Parameter</th>
<th>Value &amp; Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnets $B_{r,m}$</td>
<td>1.4 [T]</td>
</tr>
<tr>
<td>Phase current RMS</td>
<td>25 [A]</td>
</tr>
</tbody>
</table>

The SDM simulation is not accounting for the saturation effect on the magnetic flux, resulting in differences presented on the Fig. [18] the temporal harmonic content on the flux is the same but the amplitude differs on the fundamental and the first harmonic. Indeed, the saturation is flattening the spatial...
distribution of magnetic flux such that the first harmonics are lower than with SDM.

With the new conditions, the Fig. 19 and 20 respectively show the radial and tangential lumped forces and Fig. 21 the global torque. It is observable that the saturation effect is not introducing new temporal harmonics but the effect on the amplitude of the integrated quantities become obvious. In particular, the saturation is reducing the amplitude of the magnetic field in the air-gap and thus the average value of the electromagnetic torque (see Table VI). Since the SDM simulation is not accounting for the saturation, the torque (or tangential force) amplitude of the "MT Air-gap" and "MT Interface" are higher than the "MT FEMM" or the "VWP". In the end, the methods are compared in terms of amplitude differences in the logarithmic scale in the Table VII. Because of the lack of physics, the "MT Tooth Tip (SDM)" and the "MT Radial (SDM)" have been ignored to compute the values in the Table VII.

**TABLE VI**  
**GLOBAL TORQUE ON THE MACHINE WITH SATURATION**

<table>
<thead>
<tr>
<th></th>
<th>Value [N.m⁻¹]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MT Air-gap (FEA)</td>
<td>-20.8</td>
</tr>
<tr>
<td>MT Interface (FEA)</td>
<td>-17.4</td>
</tr>
<tr>
<td>VWP (FEA)</td>
<td>-23.8</td>
</tr>
<tr>
<td>MT Radial (SDM)</td>
<td>0</td>
</tr>
<tr>
<td>MT Tooth Tip (SDM)</td>
<td>-14.8</td>
</tr>
<tr>
<td>MT Air-gap (SDM)</td>
<td>-23.5</td>
</tr>
<tr>
<td>MT Interface (SDM)</td>
<td>-28.8</td>
</tr>
</tbody>
</table>

**TABLE VII**  
**MAXIMUM AMPLITUDE DIFFERENCES BETWEEN FORCE CALCULATION METHOD (IN DB SCALE)**

<table>
<thead>
<tr>
<th>Force</th>
<th>Maximum difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>Saturated</td>
</tr>
<tr>
<td>Radial</td>
<td>2.4 dB</td>
</tr>
<tr>
<td>Tangential</td>
<td>4.8 dB</td>
</tr>
<tr>
<td>Torque</td>
<td>4 dB</td>
</tr>
</tbody>
</table>
The characterization of this sensitivity is the matter of further
the geometry and the validity of the lumped force methods.
more sensitive to saturation, there is certainly a link between
weakened by the saturation. Knowing that some geometries are
assumptions of the lumped force at tooth tip method are
opening. Moreover the last part of the paper shows that the
forces if the method is correctly conserving global quantities
especially at the bottom of the slot. Then it is more convenient
gap do not properly compute the local magnetic pressure
air gap with lumping force mapping (as it is done in
standard geometries), it is very light in term of computational
cost and is as accurate as nodal methods.
The authors goal is to provide in a later paper further
investigations on a comparison in the electrical machine noise
context between nodal forces projection (which should be
more accurate) and simplified methods such as lumped force
mapping.

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